

# Two-dimensional $N=8$ supersymmetric mechanics in superspace

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## Abstract

We construct a two-dimensional  $N = 8$  supersymmetric quantum mechanics which inherits the most interesting properties of  $N = 2, d = 4$  supersymmetric Yang-Mills theory. After dimensional reduction to one dimension in terms of field-strength, we show that only complex scalar fields from the  $N = 2, d = 4$  vector multiplet become physical bosons in  $d = 1$ . The rest of the bosonic components are reduced to auxiliary fields, thus giving rise to the **(2, 8, 6)** supermultiplet in  $d = 1$ . We construct the most general superfields action for this supermultiplet and demonstrate that it possesses duality symmetry extended to the fermionic sector of theory. We also explicitly present the Dirac brackets for the canonical variables and construct the supercharges and Hamiltonian which form a  $N = 8$  super Poincaré algebra with central charges. Finally, we discuss the duality transformations which relate the **(2, 8, 6)** supermultiplet with the **(4, 8, 4)** one.

# 1 Introduction

Among extended supersymmetric theories in diverse dimensions those which have eight real supercharges are most interesting. These theories admit off-shell superfield formulations (in the harmonic [1] or projective [2] superspace) which greatly simplify their analysis. Moreover, it is possible to obtain exact quantum results for  $N = 2$ ,  $d = 4$  theories in the famous Seiberg-Witten approach [3, 4]. Finally, the theories with eight supercharges are the highest- $N$  case of theories with extended supersymmetries which have a rich geometric structure of the target space (see e.g. [5]).

One of the most investigated theories with eight supercharges is  $N = 2$ ,  $d = 4$  supersymmetric Yang-Mills (SYM) theory. It has been much explored and many exciting results have been obtained. The heart of the  $N = 2$ ,  $d = 4$  SYM theory is formed by a vector supermultiplet, which describes spin-1 particles, accompanied by complex scalar fields and doublets of spinor fields. The geometry of the scalar fields is restricted to be a Kähler one [6] of special type. The restriction that the metric be defined by a holomorphic function is crucial for the Seiberg-Witten approach. Other interesting properties of the  $N = 2$ ,  $d = 4$  SYM theory are duality in the scalar sector [3, 4] and the possibility of partial spontaneous breaking of  $N = 2$  supersymmetry by adding two types of Fayet-Iliopoulos (FI) terms [7, 8].

In [9] it has been shown that the theories with eight supercharges can be similarly formulated in diverse dimensions still sharing common properties. In this respect, the one-dimensional case has a special status, because the standard reduction from the  $N = 2$ ,  $d = 4$  SYM to  $d = 1$  gives rise to the  $N = 8$  supersymmetric theory with five bosons, i.e., the **(5, 8, 3)** supermultiplet<sup>1</sup> [9, 10]. Of course, after such a reduction almost all nice features of  $N = 2$  SYM mentioned above disappear. Naturally, an obvious question arises whether it is possible to construct an  $N = 8$ ,  $d = 1$  theory which

- contains two bosonic fields with a special Kähler geometry in the target-space,
- possesses the duality transformations, properly extended to the fermionic sector,
- may be obtained by reduction from the  $N = 2$ ,  $d = 4$  SYM,
- has a proper place for FI terms and therefore possesses non-trivial potentials.

In our paper [11] we constructed, within the Hamiltonian framework, the  $N = 8$  supersymmetric mechanics (SM) which possesses the first two properties. The goal of the present paper is to show this  $N = 8$  SM may be constructed in superspace as the direct reduction from  $N = 2$ ,  $d = 4$  SYM. The main idea is to perform the reduction to one dimension in terms of the  $N = 2$  vector multiplet  $\mathcal{A}$ , instead of the reduction in terms of a prepotential [9, 10]. In this approach only a complex scalar from the  $N = 2$ ,  $d = 4$  vector multiplet becomes a physical boson in  $d = 1$ , while the rest of the bosonic components are reduced to auxiliary fields. Thus, we end up with the **(2, 8, 6)** supermultiplet. In Subsection 2.1 we construct the most general action for this supermultiplet in terms of  $N = 8$  superfields with all possible FI terms included. We also explicitly demonstrate that the action possesses duality symmetry extended to the fermionic sector of the theory. In Subsection 2.2, in order to deal with the second-class constraints, we introduce the Dirac brackets for the canonical variables, and find the supercharges and Hamiltonian which form  $N = 8$  super Poincaré algebra with central charges. The extension of two-dimensional  $N = 8$  SM to the  $2n$ -dimensional case is performed in Subsection 2.3. Then, in Section 3 we show that the special duality transformations relate two-dimensional  $N = 8$  SM with a particular case of Quaternionic SM [12]. Finally, in Section 4 we summarize our results and draw some conclusions.

## 2 Two-dimensional N=8 SM

In this section we describe a general superfield formalism [13] and construct the most general action for two-dimensional  $N = 8$  SM. We start with the formulation of SM in  $N = 8$  superspace and conclude with the component form of the Lagrangian and Hamiltonian. The generalization of the two-dimensional case to the  $2n$ -dimensional is straightforward. So, we explicitly present here just the final results for the  $2n$ -dimensional SM. A preliminary description of part of the results was presented at a recent conference [14].

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<sup>1</sup>We use the notation **(n, N, N - n)**, in order to describe a supermultiplet with  $n$  physical bosons,  $N$  fermions and  $N - n$  auxiliary bosons.

## 2.1 Two-dimensional N=8 SM in superspace

A convenient starting point is the  $N = 8$ ,  $d = 1$  superspace  $\mathbb{R}^{(1|8)}$

$$\mathbb{R}^{(1|8)} = (t, \theta^{ia}, \vartheta^{i\alpha}), \quad (\theta^{ia})^\dagger = \theta_{ia}, \quad (\vartheta^{i\alpha})^\dagger = \vartheta_{i\alpha},$$

where  $i, a, \alpha = 1, 2$  are doublet indices of three  $SU(2)$  subgroups of the automorphism group of  $N = 8$  superspace<sup>2</sup>. In this superspace we define the covariant spinor derivatives

$$\begin{aligned} D^{ia} &= \frac{\partial}{\partial \theta_{ia}} + i\theta^{ia}\partial_t, & \nabla^{i\alpha} &= \frac{\partial}{\partial \vartheta_{i\alpha}} + i\vartheta^{i\alpha}\partial_t, \\ \{D^{ia}, D^{jb}\} &= 2i\epsilon^{ij}\epsilon^{ab}\partial_t, & \{\nabla^{i\alpha}, \nabla^{j\beta}\} &= 2i\epsilon^{ij}\epsilon^{\alpha\beta}\partial_t. \end{aligned} \quad (1)$$

In full analogy with  $N = 2, d = 4$  SYM, we introduce a complex  $N = 8$  superfield  $\mathcal{Z}$ ,  $\overline{\mathcal{Z}}$  subjected to the following constraints:

$$\begin{aligned} D^{1a}\mathcal{Z} &= \nabla^{1\alpha}\mathcal{Z} = 0, & D^{2a}\overline{\mathcal{Z}} &= \nabla^{2\alpha}\overline{\mathcal{Z}} = 0, & (a) \\ \nabla^{2\alpha}D^{2a}\mathcal{Z} + \nabla^{1\alpha}D^{1a}\overline{\mathcal{Z}} &= iM^{a\alpha}, & & & (b) \end{aligned} \quad (2)$$

where  $M^{a\alpha}$  are arbitrary constants obeying the reality condition  $(M^{a\alpha})^\dagger = M_{a\alpha}$ . The constraints (2a) represent the twisted version of the standard chirality conditions, while (2b) are recognized as modified reality constraints [8]. As we will see below, the presence of these arbitrary parameters  $M^{a\alpha}$  gives rise to potential terms in the component action and opens a possibility for a partial breaking of  $N = 8$  supersymmetry.

The constraints (2) leave the following components in the  $N = 8$  superfields  $\mathcal{Z}$ ,  $\overline{\mathcal{Z}}$ :

$$\begin{aligned} z &= \mathcal{Z}, & \bar{z} &= \overline{\mathcal{Z}}, & Y^{a\alpha} &= D^{2a}\nabla^{2\alpha}\mathcal{Z}, & \overline{Y}^{a\alpha} &= -D^{1a}\nabla^{1\alpha}\overline{\mathcal{Z}} = Y^{a\alpha} + iM^{a\alpha}, \\ \psi^a &= D^{2a}\mathcal{Z}, & \bar{\psi}_a &= -D_a^1\overline{\mathcal{Z}}, & \xi^\alpha &= \nabla^{2\alpha}\mathcal{Z}, & \bar{\xi}_\alpha &= -\nabla_\alpha^1\overline{\mathcal{Z}}, \\ A &= -iD^{2a}D_a^2\mathcal{Z}, & \bar{A} &= -iD^{1a}D_a^1\overline{\mathcal{Z}}, & B &= -i\nabla^{2\alpha}\nabla_\alpha^2\mathcal{Z}, & \bar{B} &= -i\nabla^{1\alpha}\nabla_\alpha^1\overline{\mathcal{Z}}, \end{aligned} \quad (3)$$

where the right hand side of each expression is supposed to be taken upon  $\theta^{ia} = \vartheta^{i\alpha} = 0$ . The bosonic fields  $A$  and  $B$  are subjected, in virtue of (2), to the additional constraints

$$\frac{\partial}{\partial t}(A - \bar{B}) = 0, \quad \frac{\partial}{\partial t}(\bar{A} - B) = 0. \quad (4)$$

In order to deal with these constraints, we have the following options:

- to solve them as

$$A = C + \frac{m}{2}, \quad B = \bar{C} - \frac{\bar{m}}{2}, \quad (5)$$

where  $C$  is a new independent complex auxiliary field and  $m$  is a complex constant parameter; the resulting supermultiplet will be just the **(2, 8, 6)** one;

- to insert them with Lagrange multipliers in the proper action; this option gives rise to a **(4, 8, 4)** supermultiplet and will be considered in section 3.

Thus, the direct reduction of the basic constraints defining the  $N = 2, d = 4$  vector supermultiplet to  $N = 8, d = 1$  superspace gives (with the first option selected) the **(2, 8, 6)** supermultiplet. Let us stress that the possibility to explicitly solve the constraints (4) as in (5) is the key feature of one-dimensional theories which makes such a reduction to the one-dimensional supermultiplet with only two physical bosons permissible.

Now one can write down the most general  $N = 8$  supersymmetric Lagrangian in the  $N = 8$  superspace<sup>3</sup>

$$\begin{aligned} S = - \int dt d^2\theta_2 d^2\vartheta_2 & \left[ \mathcal{F}(\mathcal{Z}) - \frac{1}{2}\theta_{2a}\vartheta_{2\alpha}N^{a\alpha}\mathcal{Z} - \frac{i}{8}(\bar{n}\theta_2^a\theta_{2a} + n\vartheta_2^\alpha\vartheta_{2\alpha})\mathcal{Z} \right] - \\ & \int dt d^2\theta_1 d^2\vartheta_1 \left[ \overline{\mathcal{F}}(\overline{\mathcal{Z}}) + \frac{1}{2}\theta_{1a}\vartheta_{1\alpha}N^{a\alpha}\overline{\mathcal{Z}} - \frac{i}{8}(n\theta_1^a\theta_{1a} + \bar{n}\vartheta_1^\alpha\vartheta_{1\alpha})\overline{\mathcal{Z}} \right]. \end{aligned} \quad (6)$$

<sup>2</sup>We use the following convention for the skew-symmetric tensor  $\epsilon$ :  $\epsilon_{ij}\epsilon^{jk} = \delta_i^k$ ,  $\epsilon_{12} = \epsilon^{21} = 1$ .

<sup>3</sup>We use the convention  $\int dt d^2\theta_2 d^2\vartheta_2 \equiv \frac{1}{16} \int dt D^{2a}D_a^2 \nabla^{2\alpha}\nabla_\alpha^2$ .

Here  $\mathcal{F}(\mathcal{Z})$  and  $\overline{\mathcal{F}}(\overline{\mathcal{Z}})$  are arbitrary holomorphic functions of the superfields  $\mathcal{Z}$  and  $\overline{\mathcal{Z}}$ , respectively, and the two terms with a constant real matrix parameter  $N^{a\alpha}$  ( $(N^{a\alpha})^\dagger = N_{a\alpha}$ ) and a complex constant parameter  $n$  represent one-dimensional versions of two FI terms [8].

Following the first option (5), after integration over the Grassmann variables and excluding the auxiliary fields  $C, Y_{a\alpha}$  by their equations of motion, we will get the action in terms of the physical components<sup>4</sup>

$$S = \int dt [\mathcal{K} - \mathcal{V}], \quad (7)$$

where the kinetic  $\mathcal{K}$  and potential  $\mathcal{V}$  terms read

$$\mathcal{K} = (F'' + \overline{F}'') \left[ \dot{z}\dot{\bar{z}} + \frac{i}{4} (\psi\dot{\bar{\psi}} - \bar{\psi}\dot{\psi} + \xi\dot{\bar{\xi}} - \bar{\xi}\dot{\xi}) \right] - \frac{i}{4} (F''' \dot{z} - \overline{F}''' \dot{\bar{z}}) (\psi\bar{\psi} + \xi\bar{\xi}) \quad (8)$$

and

$$\begin{aligned} \mathcal{V} = & \frac{1}{16} \left( F^{(4)} - \frac{3F'''F'''}{F'' + \overline{F}''} \right) \psi^2 \xi^2 + \frac{1}{16} \left( \overline{F}^{(4)} - \frac{3\overline{F}''' \overline{F}'''}{F'' + \overline{F}''} \right) \bar{\psi}^2 \bar{\xi}^2 + \\ & \frac{1}{16} \frac{F''' \overline{F}'''}{F'' + \overline{F}''} (\psi^2 \bar{\psi}^2 + \xi^2 \bar{\xi}^2 - 4\psi\bar{\psi}\xi\bar{\xi}) + \frac{1}{8} \frac{\mathcal{M}\bar{\mathcal{M}} + \mathcal{N}\bar{\mathcal{N}} + \mathcal{K}\bar{\mathcal{K}}}{F'' + \overline{F}''}, \end{aligned} \quad (9)$$

with

$$\mathcal{M} = \frac{1}{2}(F''\bar{m} - n) + iF''' \xi^2, \quad \mathcal{N} = \frac{1}{2}(F''m + \bar{n}) - iF''' \psi^2, \quad \mathcal{K}_{a\alpha} = F''M_{a\alpha} + iN_{a\alpha} + 2iF''' \psi_a \xi_\alpha. \quad (10)$$

Here the holomorphic function  $F(z)$  is defined as a bosonic limit of  $\mathcal{F}(\mathcal{Z})$

$$F(z) \equiv \mathcal{F}(\mathcal{Z})|_{\theta=\bar{\theta}=0}.$$

The action (7) is invariant with respect to the  $N = 8$  supersymmetry which is realized on the physical component fields as follows:

$$\begin{aligned} \delta z &= \epsilon_{2a} \psi^a + \epsilon_{2\alpha} \xi^\alpha, & \delta \bar{z} &= -\epsilon_{1a} \bar{\psi}^a - \epsilon_{1\alpha} \bar{\xi}^\alpha, \\ \delta \psi_a &= \frac{i}{2} \epsilon_{2a} \left( C + \frac{m}{2} \right) + \epsilon_2^a Y_{a\alpha} + 2i\epsilon_{1a} \dot{z}, & \\ \delta \xi_\alpha &= \frac{i}{2} \epsilon_{2\alpha} \left( \overline{C} - \frac{\bar{m}}{2} \right) - \epsilon_2^a Y_{a\alpha} + 2i\epsilon_{1\alpha} \dot{\bar{z}}, & \end{aligned} \quad (11)$$

$$C = \frac{\bar{\mathcal{M}} - \bar{\mathcal{N}}}{F'' + \overline{F}''}, \quad Y_{a\alpha} = -\frac{F''' \psi_a \xi_\alpha + \overline{F}''' \bar{\psi}_a \bar{\xi}_\alpha + i\mathcal{K}_{a\alpha}}{F'' + \overline{F}''}, \quad (12)$$

with  $\epsilon_{ia}, \epsilon_{i\alpha}$  being the parameters of two  $N = 4$  supersymmetries acting on  $\theta^{ia}$  and  $\vartheta^{i\alpha}$ , respectively. Using the Noether theorem one can find classical expressions for the conserved supercharges corresponding to the supersymmetry transformations (11)

$$\begin{aligned} Q_1^a &= (F'' + \overline{F}'') \psi^a \dot{\bar{z}} - \frac{i}{4} \overline{F}^{(3)} \bar{\psi}^a \bar{\xi}^2 - \frac{1}{2} (\overline{F}'' M^{a\alpha} - iN^{a\alpha}) \bar{\xi}_\alpha - \frac{1}{4} (\overline{F}'' m - \bar{n}) \bar{\psi}^a, \\ S_1^\alpha &= (F'' + \overline{F}'') \xi^\alpha \dot{\bar{z}} - \frac{i}{4} \overline{F}^{(3)} \bar{\xi}^\alpha \bar{\psi}^2 + \frac{1}{2} (\overline{F}'' M^{a\alpha} - iN^{a\alpha}) \bar{\psi}_a + \frac{1}{4} (\overline{F}'' \bar{m} + n) \bar{\xi}^\alpha, \\ Q_{2a} &= (Q_1^a)^\dagger, \quad S_{2a} = (S_1^a)^\dagger. \end{aligned} \quad (13)$$

Let us stress, once again, that our variant of  $N = 8$  SM is a reduction of  $N = 2, d = 4$  SYM. So it is not unexpected that the metric of the bosonic manifold is restricted to be the *special Kähler* one (of rigid type)

$$g(z, \bar{z}) = F''(z) + \overline{F}''(\bar{z}). \quad (14)$$

Moreover, one may immediately check that the action (7) exhibits the famous Seiberg-Witten duality [3] extended to the fermionic sector of theory. Indeed, after passing to new variables defined as

$$\begin{aligned} z &\rightarrow iF'(z), & \bar{z} &\rightarrow -i\overline{F}'(\bar{z}), & F''(z) &\rightarrow 1/F''(z), & \overline{F}''(\bar{z}) &\rightarrow 1/\overline{F}''(\bar{z}), \\ \psi^a &\rightarrow iF''(z)\psi^a, & \bar{\psi}_a &\rightarrow -i\overline{F}''(\bar{z})\bar{\psi}_a, & \xi^\alpha &\rightarrow iF''(z)\xi^\alpha, & \bar{\xi}_\alpha &\rightarrow -i\overline{F}''(\bar{z})\bar{\xi}_\alpha, \\ N^{a\alpha} &\rightarrow M^{a\alpha}, & M^{a\alpha} &\rightarrow -N^{a\alpha}, & m &\rightarrow \bar{n}, & n &\rightarrow \bar{m}, \end{aligned} \quad (15)$$

<sup>4</sup>All implicit summations go from “up-left” to “down-right”, e.g.,  $\psi\bar{\psi} \equiv \psi^a \bar{\psi}_a$ ,  $\psi^2 \equiv \psi^a \psi_a$ ,  $M^2 \equiv M^{a\alpha} M_{a\alpha}$ , etc.

the action (7) keeps its form. Let us note that in the dual formulation the constants  $M^{a\alpha}$  and  $m$ , which appear in the constraints (2) and (5), are interchanged with the constants  $N^{a\alpha}$  and  $n$ , which have shown up in the FI terms. This is just a simplified version of the electric-magnetic duality [3] for our  $N = 8$  SM case. Thus, our  $N = 8$  SM possesses the most interesting peculiarities of the  $N = 2$ ,  $d = 4$  SYM theory and can be used for a simplified analysis of some subtle properties of its ancestor.

## 2.2 Two-dimensional N=8 SM: Hamiltonian

In order to find the classical Hamiltonian, we follow the standard procedure of quantizing a system with bosonic and fermionic degrees of freedom. From the action (7) we define the momenta  $p$ ,  $\bar{p}$ ,  $\pi_a^{(\psi)}$ ,  $\bar{\pi}^{(\psi)a}$ ,  $\pi_\alpha^{(\xi)}$ ,  $\bar{\pi}^{(\xi)\alpha}$  conjugated to  $z$ ,  $\bar{z}$ ,  $\psi^a$ ,  $\bar{\psi}_a$ ,  $\xi^\alpha$  and  $\bar{\xi}_\alpha$ , respectively, as

$$\begin{aligned} p &= g\dot{z} - \frac{i}{4}\partial_z g (\psi\bar{\psi} + \xi\bar{\xi}), & \bar{p} &= g\dot{\bar{z}} + \frac{i}{4}\bar{\partial}_z g (\psi\bar{\psi} + \xi\bar{\xi}), \\ \pi_a^{(\psi)} &= -\frac{i}{4}g\dot{\bar{\psi}}_a, & \bar{\pi}^{(\psi)a} &= -\frac{i}{4}g\dot{\psi}^a, & \pi_\alpha^{(\xi)} &= -\frac{i}{4}g\dot{\bar{\xi}}_\alpha, & \bar{\pi}^{(\xi)\alpha} &= -\frac{i}{4}g\dot{\xi}^\alpha, \end{aligned} \quad (16)$$

with the metric  $g(z, \bar{z})$  defined in (14) and introduce Dirac brackets for the canonical variables

$$\begin{aligned} \{z, \hat{p}\} &= 1, & \{\bar{z}, \hat{\bar{p}}\} &= 1, & \{\hat{p}, \hat{\bar{p}}\} &= -\frac{i}{2}\frac{\partial_z g \bar{\partial}_z g}{g} (\psi\bar{\psi} + \xi\bar{\xi}), \\ \{\hat{p}, \psi_a\} &= \frac{\partial_z g}{g} \psi_a, & \{\hat{\bar{p}}, \bar{\psi}_a\} &= \frac{\bar{\partial}_z g}{g} \bar{\psi}_a, & \{\psi^a, \bar{\psi}_b\} &= -\frac{2i}{g} \delta_b^a, \\ \{\hat{p}, \xi_\alpha\} &= \frac{\partial_z g}{g} \xi_\alpha, & \{\hat{\bar{p}}, \bar{\xi}_\alpha\} &= \frac{\bar{\partial}_z g}{g} \bar{\xi}_\alpha, & \{\xi^\alpha, \bar{\xi}_\beta\} &= -\frac{2i}{g} \delta_\beta^\alpha, \end{aligned} \quad (17)$$

where the “improved” bosonic momenta have been defined as

$$\hat{p} \equiv p + \frac{i}{4}\partial_z g (\psi\bar{\psi} + \xi\bar{\xi}), \quad \hat{\bar{p}} \equiv \bar{p} + \frac{i}{4}\bar{\partial}_z g (\psi\bar{\psi} + \xi\bar{\xi}). \quad (18)$$

Now one can check that the supercharges  $Q_{ia}$ ,  $S_{i\alpha}$  (13), being rewritten through the momenta as

$$\begin{aligned} Q_1^a &= \hat{p} \psi^a - \frac{i}{4}\bar{\partial}_z g \bar{\psi}^a \bar{\xi}^2 - \frac{1}{2}(\bar{F}''' M^{a\alpha} - iN^{a\alpha}) \bar{\xi}_\alpha - \frac{1}{4}(\bar{F}'' m - \bar{n}) \bar{\psi}^a, \\ S_1^\alpha &= \hat{p} \xi^\alpha - \frac{i}{4}\bar{\partial}_z g \bar{\xi}^\alpha \bar{\psi}^2 + \frac{1}{2}(\bar{F}''' M^{a\alpha} - iN^{a\alpha}) \bar{\psi}_a + \frac{1}{4}(\bar{F}'' \bar{m} + n) \bar{\xi}^\alpha, \\ Q_{2a} &= (Q_1^a)^\dagger, \quad S_{2\alpha} = (S_1^\alpha)^\dagger, \end{aligned} \quad (19)$$

and the Hamiltonian

$$H = g^{-1} \hat{p} \hat{\bar{p}} + \mathcal{V} \quad (20)$$

form the  $N = 8$  Poincaré superalgebra with the central charges

$$\begin{aligned} \{Q_{ia}, Q_{jb}\} &= -2i\epsilon_{ij}\epsilon_{ab} \left( H - \frac{1}{16}(nm + \bar{n}\bar{m}) \right) - \frac{1}{8}\epsilon_{ij} (N_a^\alpha M_{ab} + N_b^\alpha M_{a\alpha}), \\ \{S_{i\alpha}, S_{j\beta}\} &= -2i\epsilon_{ij}\epsilon_{\alpha\beta} \left( H + \frac{1}{16}(nm + \bar{n}\bar{m}) \right) - \frac{1}{8}\epsilon_{ij} (N_\alpha^a M_{a\beta} + N_\beta^a M_{a\alpha}), \\ \{Q_{1a}, S_{2\alpha}\} &= -mN_{a\alpha} - i\bar{n}M_{a\alpha}, \quad \{Q_{2a}, S_{1\alpha}\} = -\bar{m}N_{a\alpha} + inM_{a\alpha}. \end{aligned} \quad (21)$$

By these we complete the classical description of the two-dimensional  $N = 8$  SM.

Before closing this Subsection and going on to generalize our SM to the  $2n$ -dimensional case, let us briefly summarize the main peculiarities of the model.

Firstly, as it has already been mentioned, the  $N = 8$  supersymmetry strictly fixes the metric of the target space to be the *special Kähler* one.

Next, the presence of the central charges in the superalgebra (21), as in the  $N = 4$  SM case [15], is the most exciting feature of the model. The central charges appear only when the FI terms are added (with the constants  $N_{a\alpha}$  or  $n$ ) and the auxiliary fields contain the constant parts ( $M_{a\alpha}$  or  $m$ ). The existence of the nonzero central charges in the superalgebra (21) opens up the possibility of realizing a partial spontaneous breaking of  $N = 8$  supersymmetry.

Finally, it is worth noticing that the bosonic potential terms, which appear in the Hamiltonian, explicitly break at least one of the  $SU(2)$  automorphism groups. This feature is, once again, very similar to the case of  $N = 4$  SM [15].

### 2.3 2n-dimensional N=8 SM

The generalization of the  $N = 8$  two-dimensional SM to the  $2n$ -dimensional case is straightforward. The simplest one is the superfield generalization. The related steps are described in the following.

- We introduce  $n$  complex  $N = 8$  superfields  $\mathcal{Z}^A, \bar{\mathcal{Z}}^B$  ( $A, B = 1, \dots, n$ ), each of them obeys the same constraints (2) with different constants  $M^{Aa\alpha}$

$$\begin{aligned} D^{1a} \mathcal{Z}^A &= \nabla^{1\alpha} \mathcal{Z}^A = 0, & D^{2a} \bar{\mathcal{Z}}^A &= \nabla^{2\alpha} \bar{\mathcal{Z}}^A = 0, & (a) \\ \nabla^{2\alpha} D^{2a} \mathcal{Z}^A + \nabla^{1\alpha} D^{1a} \bar{\mathcal{Z}}^A &= iM^{Aa\alpha}. & (b) \end{aligned} \quad (22)$$

- The components of each superfield can be defined as in (3) and  $n$  different constants  $m^A$  may be introduced similarly to (5)

$$A^A = C^A + \frac{m^A}{2}, \quad B^A = \bar{C}^A - \frac{\bar{m}^A}{2}. \quad (23)$$

- The most general  $N = 8$  supersymmetric action reads

$$\begin{aligned} S_{2n} = & - \int dt d^2 \theta_2 d^2 \vartheta_2 \left[ \mathcal{F}(\mathcal{Z}^1, \dots, \mathcal{Z}^k) - \frac{1}{2} \theta_{2a} \vartheta_{2\alpha} \sum_A N_A^{a\alpha} \mathcal{Z}^A - \right. \\ & \left. \frac{i}{8} \sum_A (\bar{n}_A \theta_2^a \theta_{2a} + n_A \vartheta_2^\alpha \vartheta_{2\alpha}) \mathcal{Z}^A \right] + c.c., \end{aligned} \quad (24)$$

where  $\mathcal{F}(\mathcal{Z}^1, \dots, \mathcal{Z}^k), \bar{\mathcal{F}}(\bar{\mathcal{Z}}^1, \dots, \bar{\mathcal{Z}}^k)$  are arbitrary holomorphic functions of the  $n$  superfields  $\mathcal{Z}^A$  and  $\bar{\mathcal{Z}}^A$ , respectively, and all possible FI terms with the constants  $N_A^{a\alpha}$  and  $n_A$  are included.

The rest of the calculations goes in the same way as it is done in the previous subsections. For completeness, we present here the explicit structure of the Dirac brackets between the canonical variables

$$\begin{aligned} \{z^A, \hat{p}_B\} &= \delta_B^A, & \{\bar{z}^A, \hat{\bar{p}}_B\} &= \delta_B^A, \\ \{\hat{p}_A, \hat{\bar{p}}_B\} &= -\frac{i}{2} g^{EE'} \partial_{ACE}^3 F \bar{\partial}_{BC'E'}^3 \bar{F} \left( \psi^a C \bar{\psi}_a^{C'} + \xi^{\alpha C} \bar{\xi}_\alpha^{C'} \right), \\ \{\psi^{Aa}, \bar{\psi}_b^B\} &= -2ig^{AB} \delta_b^a, & \{\xi^{A\alpha}, \bar{\xi}_\beta^B\} &= -2ig^{AB} \delta_\beta^\alpha, \\ \{\hat{p}_A, \psi_a^B\} &= g^{BC} \partial_{ACE}^3 F \psi_a^E, & \{\hat{p}_A, \xi_\alpha^B\} &= g^{BC} \partial_{ACE}^3 F \xi_\alpha^E, \\ \{\hat{\bar{p}}_A, \bar{\psi}_a^B\} &= g^{BC} \bar{\partial}_{ACE}^3 \bar{F} \bar{\psi}_a^E, & \{\hat{\bar{p}}_A, \bar{\xi}_\alpha^B\} &= g^{BC} \bar{\partial}_{ACE}^3 \bar{F} \bar{\xi}_\alpha^E, \end{aligned} \quad (25)$$

where the metric  $g_{AB}$  and its inverse  $g^{AB}$  are defined as

$$g_{AB} = \frac{\partial^2}{\partial z^A \partial \bar{z}^B} F(z^1, \dots, z^k) + \frac{\partial^2}{\partial \bar{z}^A \partial z^B} \bar{F}(\bar{z}^1, \dots, \bar{z}^k), \quad g^{AB} g_{BC} = \delta_C^A. \quad (26)$$

Like to the previous section it is convenient to introduce  $n$ -dimensional extension of the quantities (10)

$$\begin{aligned} \mathcal{M}_A &= \frac{1}{2} (\partial_{AB}^2 F \bar{m}^B - n_A) + i \partial_{ABC}^3 F \xi^{B\alpha} \xi_\alpha^C, & \mathcal{N}_A &= \frac{1}{2} (\partial_{AB}^2 F m^B + \bar{n}_A) - i \partial_{ABC}^3 F \psi^{Ba} \psi_a^C, \\ \mathcal{K}_A^{a\alpha} &= \partial_{AB}^2 F M^{Ba\alpha} + i N_A^{a\alpha} + 2i \partial_{ABC}^3 F \psi^{Ba} \xi^{C\alpha} \end{aligned} \quad (27)$$

to have the supercharges and Hamiltonian been written in term of these objects quite elegantly. The supercharges

$$\begin{aligned} Q_1^a &= \hat{p}_A \psi^{Aa} - \frac{i}{4} \bar{\partial}_{ABC}^3 \bar{F} \bar{\psi}^{Aa} \bar{\xi}^{B\alpha} \bar{\xi}_\alpha^C - \frac{1}{2} (\bar{\partial}_{AB}^2 \bar{F} M^{Ba\alpha} - i N_A^{a\alpha}) \bar{\xi}_\alpha^A - \frac{1}{4} (\bar{\partial}_{AB}^2 \bar{F} m^B - \bar{n}_A) \bar{\psi}^{Aa}, \\ S_1^\alpha &= \hat{p}_A \xi^{A\alpha} - \frac{i}{4} \bar{\partial}_{ABC}^3 \bar{F} \bar{\xi}^{A\alpha} \bar{\psi}^{Ba} \bar{\psi}_a^C + \frac{1}{2} (\bar{\partial}_{AB}^2 \bar{F} M^{Ba\alpha} - i N_A^{a\alpha}) \bar{\psi}_a^A + \frac{1}{4} (\bar{\partial}_{AB}^2 \bar{F} \bar{m}^B + n_A) \bar{\xi}^{A\alpha}, \\ Q_{2a} &= (Q_1^a)^\dagger, & S_{2a} &= (S_1^a)^\dagger \end{aligned} \quad (28)$$

and the Hamiltonian

$$\begin{aligned}
H_{2n} = & g^{AB} \hat{p}_A \hat{p}_B + \\
& \frac{1}{16} \left( \partial_{ABCD}^4 F - g^{EE'} \partial_{ABE}^3 F \partial_{CDE'}^3 F - 2g^{EE'} \partial_{ACE}^3 F \partial_{BDE'}^3 F \right) \psi^{Aa} \psi_a^B \xi^{C\alpha} \xi_\alpha^D + \\
& \frac{1}{16} \left( \bar{\partial}_{ABCD}^4 \bar{F} - g^{EE'} \bar{\partial}_{ABE}^3 \bar{F} \bar{\partial}_{CDE'}^3 \bar{F} - 2g^{EE'} \bar{\partial}_{ACE}^3 \bar{F} \bar{\partial}_{BDE'}^3 \bar{F} \right) \bar{\psi}^{Aa} \bar{\psi}_a^B \bar{\xi}^{C\alpha} \bar{\xi}_\alpha^D + \\
& \frac{1}{16} g^{EE'} \partial_{ABE}^3 F \bar{\partial}_{CDE'}^3 \bar{F} \left( \psi^{Aa} \psi_a^B \bar{\psi}^{Cb} \bar{\psi}_b^D + \xi^{A\alpha} \xi_\alpha^B \bar{\xi}^{C\beta} \bar{\xi}_\beta^D - 4\psi^{Aa} \bar{\psi}_a^C \xi^{B\alpha} \bar{\xi}_\alpha^D \right) + \\
& \frac{1}{8} g^{AB} [\mathcal{M}_A \bar{\mathcal{M}}_B + \mathcal{N}_A \bar{\mathcal{N}}_B + \mathcal{K}_A^{a\alpha} \bar{\mathcal{K}}_{Ba\alpha}]
\end{aligned} \tag{29}$$

form the superalgebra

$$\begin{aligned}
\{Q_{ia}, Q_{jb}\} &= -2i\epsilon_{ij}\epsilon_{ab} \left( H - \frac{1}{16}(n_A m^A + \bar{n}_A \bar{m}^A) \right) - \frac{1}{8}\epsilon_{ij} \left( N_{Aa}^\alpha M_{\alpha b}^A + N_{Ab}^\alpha M_{\alpha a}^A \right), \\
\{S_{i\alpha}, S_{j\beta}\} &= -2i\epsilon_{ij}\epsilon_{\alpha\beta} \left( H + \frac{1}{16}(n_A m^A + \bar{n}_A \bar{m}^A) \right) - \frac{1}{8}\epsilon_{ij} \left( N_{A\alpha}^a M_{a\beta}^A + N_{A\beta}^a M_{a\alpha}^A \right), \\
\{Q_{1a}, S_{2\alpha}\} &= -m^A N_{Aa\alpha} - i\bar{n}_A M_{a\alpha}^A, \quad \{Q_{2a}, S_{1\alpha}\} = -\bar{m}^A N_{Aa\alpha} + i n_A M_{a\alpha}^A.
\end{aligned} \tag{30}$$

### 3 (4, 8, 4) supermultiplet

In this Section we discuss the duality transformations which relate the (2, 8, 6) supermultiplet with the (4, 8, 4) one.

As it has already been mentioned in Subsection 2.1 there is the other possibility to deal with the constraints (4). Namely, one can incorporate them into the action using the Lagrange multipliers  $\varphi, \bar{\varphi}$

$$\begin{aligned}
S = & \int dt \left\{ (F'' + \bar{F}'') \left[ \dot{z}\dot{\bar{z}} + \frac{i}{4} (\psi\dot{\bar{\psi}} - \dot{\psi}\bar{\psi} + \xi\dot{\bar{\xi}} - \dot{\xi}\bar{\xi}) \right] - \frac{i}{4} (F^{(3)}\dot{z} - \bar{F}^{(3)}\dot{\bar{z}}) (\psi\bar{\psi} + \xi\bar{\xi}) + \right. \\
& \frac{1}{16} (F'' (2Y^2 + AB) + \bar{F}'' (2\bar{Y}^2 + \bar{A}\bar{B}) - F^{(4)}\psi^2\xi^2 - \bar{F}^{(4)}\bar{\psi}^2\bar{\xi}^2 - \\
& F^{(3)}(iA\xi^2 + iB\psi^2 - 4\psi^a\xi^\alpha Y_{a\alpha}) - \bar{F}^{(3)}(i\bar{A}\bar{\xi}^2 + i\bar{B}\bar{\psi}^2 + 4\bar{\psi}^a\bar{\xi}^\alpha Y_{a\alpha})) + \\
& \left. \frac{1}{4}\varphi(\dot{A} - \dot{\bar{B}}) + \frac{1}{4}\bar{\varphi}(\dot{\bar{A}} - \dot{B}) \right\}.
\end{aligned} \tag{31}$$

For the sake of simplicity we consider the action (6) with all constant parameters equal to zero. Eliminating the auxiliary fields  $A, B$  and  $Y_{a\alpha}$  via their equations of motion

$$A = \frac{iF^{(3)}\psi^2 - 4\dot{\varphi}}{F''}, \quad B = \frac{iF^{(3)}\xi^2 + 4\dot{\varphi}}{F''}, \quad Y_{a\alpha} = \frac{\bar{F}^{(3)}\bar{\psi}_a\bar{\xi}_\alpha - F^{(3)}\psi_a\xi_\alpha}{F'' + \bar{F}''} \tag{32}$$

one gets the following expressions for the kinetic

$$\begin{aligned}
\mathcal{K} = & (F'' + \bar{F}'') \left[ \dot{z}\dot{\bar{z}} + \frac{i}{4} (\psi\dot{\bar{\psi}} - \dot{\psi}\bar{\psi} + \xi\dot{\bar{\xi}} - \dot{\xi}\bar{\xi}) \right] + \left( \frac{1}{F''} + \frac{1}{\bar{F}''} \right) \dot{\varphi}\dot{\bar{\varphi}} - \\
& - \frac{i}{4} (F''' \dot{z} - \bar{F}''' \dot{\bar{z}}) (\psi\bar{\psi} + \xi\bar{\xi}) - \frac{i}{4} \dot{\varphi} \left( \frac{F'''}{F''} \psi^2 - \frac{\bar{F}'''}{\bar{F}''} \bar{\xi}^2 \right) - \frac{i}{4} \dot{\bar{\varphi}} \left( \frac{\bar{F}'''}{\bar{F}''} \bar{\psi}^2 - \frac{F'''}{F''} \xi^2 \right)
\end{aligned} \tag{33}$$

and potential

$$\begin{aligned}
\mathcal{V} = & \frac{1}{16} \left( F^{(4)} - \frac{3F'''F'''}{F'' + \bar{F}''} - \frac{F'''^2\bar{F}''}{(F'' + \bar{F}'')F''} \right) \psi^2\xi^2 + \\
& + \frac{1}{16} \left( \bar{F}^{(4)} - \frac{3\bar{F}'''\bar{F}'''}{F'' + \bar{F}''} - \frac{\bar{F}'''^2F''}{(F'' + \bar{F}'')\bar{F}''} \right) \bar{\psi}^2\bar{\xi}^2 + \frac{F'''\bar{F}'''}{4(F'' + \bar{F}'')} \psi\bar{\psi}\xi\bar{\xi}
\end{aligned} \tag{34}$$

terms of the action (31). From Eq.(33) one sees that the field  $\varphi$  becomes dynamical, giving rise to SM with four physical bosons.

The possibility to invert the auxiliary fields to the physical ones in one dimension was noticed many years ago in [16]. The simplest way to demonstrate this is to consider the transformation properties of the auxiliary fields  $A$  and  $B$  which read

$$\delta A = 4\epsilon_2^\alpha \dot{\xi}_\alpha + 4\epsilon_1^a \dot{\psi}_a, \quad \delta B = 4\epsilon_2^a \dot{\bar{\psi}}_a + 4\epsilon_1^\alpha \dot{\xi}_\alpha. \quad (35)$$

Due to the fact that the r.h.s. of (35) is a total time derivative, one can make the auxiliary fields “dynamical” by setting

$$A = \bar{B} = 4\dot{\Phi}, \quad \bar{A} = B = 4\dot{\bar{\Phi}}. \quad (36)$$

The new dynamical fields  $\Phi$  and  $\bar{\Phi}$  transform under  $N = 8$  supersymmetry as

$$\delta\Phi = \epsilon_2^\alpha \bar{\xi}_\alpha + \epsilon_1^a \psi_a, \quad \delta\bar{\Phi} = \epsilon_2^a \bar{\psi}_a + \epsilon_1^\alpha \xi_\alpha \quad (37)$$

and form, with the remaining components from (3), the **(4,8,4)** supermultiplet. Substituting (36) into the action (31) one can get

$$\begin{aligned} \mathcal{K} = & (F'' + \bar{F}'') \left[ \dot{z}\dot{\bar{z}} + \dot{\Phi}\dot{\bar{\Phi}} + \frac{i}{4} \left( \psi\dot{\bar{\psi}} - \dot{\psi}\bar{\psi} + \xi\dot{\bar{\xi}} - \dot{\xi}\bar{\xi} \right) \right] - \\ & - \frac{i}{4} (F''' \dot{z} - \bar{F}''' \dot{\bar{z}}) (\psi\bar{\psi} + \xi\bar{\xi}) - \frac{i}{4} \dot{\Phi} (F''' \xi^2 + \bar{F}''' \bar{\psi}^2) - \frac{i}{4} \dot{\bar{\Phi}} (F''' \psi^2 + \bar{F}''' \bar{\xi}^2), \end{aligned} \quad (38)$$

$$\mathcal{V} = \frac{1}{16} \left( F^{(4)} - \frac{2F'''F'''}{F'' + \bar{F}''} \right) \psi^2 \xi^2 + \frac{1}{16} \left( \bar{F}^{(4)} - \frac{2\bar{F}''' \bar{F}'''}{F'' + \bar{F}''} \right) \bar{\psi}^2 \bar{\xi}^2 + \frac{F''' \bar{F}'''}{4(F'' + \bar{F}'')} \psi\bar{\psi}\xi\bar{\xi}. \quad (39)$$

An amazing fact is that the actions corresponding to Eqs.(33), (34) and Eqs.(38), (39) are transformed into each other by the same Seiberg-Witten duality transformations (15) (with the constants  $M$ ,  $N$ ,  $m$  and  $n$  omitted) augmented by

$$\varphi \rightarrow i\bar{\Phi}, \quad \bar{\varphi} \rightarrow -i\Phi, \quad \Phi \rightarrow i\bar{\varphi}, \quad \bar{\Phi} \rightarrow -i\varphi. \quad (40)$$

Thus we conclude, that two-dimensional  $N = 8$  SM is dual to  $N = 8$  Quaternionic SM [12] with the additional restriction on the target space metric to be a special Kähler one depending only on two fields  $z, \bar{z}$ .

## 4 Summary and conclusions

In the present paper we give a superfield description of  $N = 8$  SM with **(2, 8, 6)** supermultiplet. This supermultiplet is obtained by a direct reduction from the  $N = 2$ ,  $d = 4$  vector supermultiplet. We constructed the most general action in  $N = 8$  superspace with all possible FI terms and explicitly showed that the geometry of the target space is restricted to be the special Kähler one. Apart from the  $N = 8$  superfield formulation, we presented the component action with all auxiliary fields, as well as with the physical fields only. As a nice feature, the constructed action possesses a duality which acts not only in the bosonic sector, but also in the fermionic one. We performed the Hamiltonian analysis and found the Dirac brackets between the canonical variables. The supercharges and Hamiltonian form a  $N = 8$  super Poincaré algebra with central charges. The latter are proportional to the product of two constants — one that comes from the FI terms, and the other that appears in the superfield constraints. These constants are directly related to the appearance of potential terms in the Hamiltonian. We also presented the extension of the  $N = 8$  two-dimensional SM to the  $2n$ -dimensional case. The possibility to revert auxiliary fields to physical ones has been used, in order to construct a dual version of  $N = 8$  SM, which is just a special case of  $N = 8$  Quaternionic SM.

These results should be regarded as preparatory for a more detailed study of  $2n$ -dimensional SM with  $N = 8$  supersymmetry. In particular, it would be interesting to construct the full quantum version with some specific Kähler potential. Generally speaking, we believe that just this version of SM could be rather useful for a simplified analysis of subtle problems which appear in the  $N = 2$ ,  $d = 4$  SYM. For example, one may try to fully analyse the effects of non-anti-commutativity in superspace [17], including modifications of the spectra, etc.

Finally, due to the appearance of the central charges in the  $N = 8$  Poincaré superalgebra one may expect the existence of different patterns of partial supersymmetry breaking, like in the  $N = 4$  SM case [15, 18].



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# References

- [1] A.S. Galperin, E.A. Ivanov, S. Kalitzin, V.I. Ogievetsky, E.S. Sokatchev: *Class. Quantum Grav.* **1** (1984) 469;  
A.S. Galperin, E.A. Ivanov, V.I. Ogievetsky, E.S. Sokatchev: *Harmonic Superspace*. Cambridge University Press, Cambridge 2001.
- [2] A. Karlhede, U. Lindström, M. Roček: *Phys. Lett. B* **147** (1984) 297;  
U. Lindström, M. Roček: *Commun. Math. Phys.* **115** (1988) 21; *ibid.* **128** (1990) 191.
- [3] N. Seiberg, E. Witten: *Nucl. Phys. B* **426** (1994) 19; Erratum-*ibid.* **B430** (1994) 485; [hep-th/9407087](#).
- [4] N. Seiberg, E. Witten: *Nucl. Phys. B* **431** (1994) 484; [hep-th/9408099](#).
- [5] A. Van Proeyen: “*Vector multiplets in  $N = 2$  supersymmetry and its associated moduli spaces*”, Lectures given in the 1995 Trieste summer school in high energy physics and cosmology; [hep-th/9512139](#).
- [6] G. Sierra, P.K. Townsend: in “*Supersymmetry and Supergravity*”, ed. B. Milewski (World Scientific, Singapore, 1983);  
S.J. Gates: *Nucl. Phys. B* **238** (1984) 349.
- [7] I. Antoniadis, H. Partouche, T.R. Taylor: *Phys. Lett. B* **372** (1996) 83;  
S. Ferrara, L. Girardello, M. Porrati: *Phys. Lett. B* **376** (1996) 275.
- [8] E. Ivanov, B. Zupnik: *Phys. Atom. Nucl.* **62** (1999) 1043; *Yad.Fiz.* **62** (1999) 1110; [hep-th/9710236](#).
- [9] B. Zupnik: *Nucl. Phys. B* **554** (1999) 365, Erratum-*ibid.* **B 644** (2002) 405; [hep-th/9902038](#).
- [10] D.-E. Diaconescu, R. Entin: *Phys. Rev. D* **56** (1997) 8045; [hep-th/9706059](#).
- [11] S. Bellucci, S. Krivonos, A. Nersessian: *Phys. Lett. B* **605** (2005) 181; [hep-th/0410029](#).
- [12] S. Bellucci, S. Krivonos, A. Sutulin, *Phys. Lett. B* **605** (2005) 406; [hep-th/0410276](#).
- [13] S. Bellucci, E. Ivanov, S. Krivonos, O. Lechtenfeld: *Nucl. Phys. B* **699** (2004) 226; [hep-th/0406015](#);  
S. Bellucci, E. Ivanov, S. Krivonos, O. Lechtenfeld: *Nucl. Phys. B* **684** (2004) 321; [hep-th/0312322](#).
- [14] S. Bellucci, S. Krivonos, A. Nersessian, A. Shcherbakov: “*2k-dimensional  $N=8$  supersymmetric quantum mechanics*”, [hep-th/0410073](#).
- [15] E. Ivanov, S. Krivonos, A. Pashnev: *Class. Quantum. Grav.* **8** (1991) 19.
- [16] S.J. Gates, Jr., L. Rana, *Phys. Lett. B* **342** (1995) 132, [hep-th/9410150](#).
- [17] E. Ivanov, O. Lechtenfeld, B. Zupnik: *JHEP* **0402** (2004) 012; [hep-th/0308012](#);  
S. Ferrara, E. Sokatchev: *Phys. Lett. B* **579** (2004) 226; [hep-th/0308021](#).
- [18] E. Donets, A. Pashnev, J. Juan Rosales, M. Tsulaia: *Phys. Rev. D* **61** (2000) 043512; [hep-th/9907224](#).